Eur. Phys. J. C 50, 579–584 (2007)
DOI 10.1140/epjc/s10052-007-0242-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

CP violation for neutral charmed meson decays into CP eigenstates[a]

Dong-Sheng Du[b]

Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918 (4), Beijing 100049, P.R. China

Received: 16 October 2006 / Revised version: 15 January 2007 /
Published online: 6 March 2007 –

Abstract. The CP asymmetries for the decays of the neutral charmed meson into CP eigenstates are carefully studied. Formulas and numerical results are presented. The impact on experiments is briefly discussed.

1 Introduction

Up to now, we still do not have any experimental evidence for CP violation in the charm sector. Theoretically, the prediction for charm mixing in the standard model is very small [1–5]. This leads to small CP violating effects in the charm decays. However, searching for large mixing and CP violation in charm decays is still very interesting; not only for testing the standard model, but also for finding new physics. For a recent review, see [6]. Because the CP eigenstates are very special, if the D^0–$\overline{D^0}$ decay into the same CP eigenstates, then the CP violating asymmetry could be enhanced by interference. So we try to investigate this possibility in detail. Another advantage of the CP eigenstates is that the amplitude ratio $A(\overline{D^0} \to f)/A(D^0 \to f)$ can be estimated without computing the amplitudes directly. This makes the computation of the CP asymmetries easier. In this paper, we shall concentrate on the case of the CP eigenstates into which the charm decays.

2 Time-dependent CP asymmetry

Define [7]

$$\begin{aligned} CP|D^0\rangle &= |\overline{D^0}\rangle , \\ |D_\mathrm{S}\rangle &= p|D^0\rangle + q|\overline{D^0}\rangle , \\ |D_\mathrm{L}\rangle &= p|D^0\rangle - q|\overline{D^0}\rangle , \\ |p|^2+|q|^2 &= 1 . \end{aligned} \tag{1}$$

The corresponding eigenvalues of $|D_\mathrm{S}\rangle$, $|D_\mathrm{L}\rangle$ are

$$\lambda_\mathrm{S} = m_\mathrm{S} - \mathrm{i}\frac{\gamma_\mathrm{S}}{2} , \quad \lambda_\mathrm{L} = m_\mathrm{L} - \mathrm{i}\frac{\gamma_\mathrm{L}}{2} .$$

Assuming CPT invariance, the time-evolved states are

$$\begin{aligned} |D_p^0(t)\rangle &= g_+(t)|D^0\rangle + \frac{q}{p} g_-(t)|\overline{D^0}\rangle , \\ |\overline{D_p^0}(t)\rangle &= \frac{p}{q} g_-(t)|D^0\rangle + g_+(t)|\overline{D^0}\rangle , \end{aligned} \tag{2}$$

where

$$\begin{cases} g_\pm = \dfrac{1}{2}(\mathrm{e}^{-\mathrm{i}\lambda_\mathrm{S} t} \pm \mathrm{e}^{-\mathrm{i}\lambda_\mathrm{L} t}) \\ \quad = \dfrac{1}{2}\mathrm{e}^{-\mathrm{i}mt-\frac{\gamma}{2}t}\left\{\mathrm{e}^{\mathrm{i}\frac{\Delta m}{2}t-\frac{\Delta\gamma}{4}t} \pm \mathrm{e}^{-\mathrm{i}\frac{\Delta m}{2}+\frac{\Delta\gamma}{4}t}\right\} , \\ \Delta m = m_\mathrm{L} - m_\mathrm{S} , \quad m = (m_\mathrm{L}+m_\mathrm{S})/2 , \\ \Delta\gamma = \gamma_\mathrm{S} - \gamma_\mathrm{L} , \quad \gamma = (\gamma_\mathrm{L}+\gamma_\mathrm{S})/2 . \end{cases} \tag{3}$$

Define the mixing parameter

$$x = \frac{\Delta m}{\gamma} , \quad y = \frac{\Delta\gamma}{2\gamma} ; \tag{4}$$

then the decay amplitudes for the final state f are

$$\begin{aligned} A\big(D_p^0(t) \to f\big) &= \big\langle f\big|H_\mathrm{eff}\big|D_p^0(t)\big\rangle \\ &= g_+(t)A(f) + \frac{q}{p} g_-(t)\bar{A}(f) \\ &= A(f)\{g_+(t) + \lambda_f g_-(t)\} , \end{aligned} \tag{5}$$

where

$$\begin{aligned} A(f) &= \big\langle f\big|H_\mathrm{eff}\big|D^0\big\rangle , \\ \bar{A}(f) &= \big\langle f\big|H_\mathrm{eff}\big|\overline{D^0}\big\rangle , \\ \lambda_f &= \frac{q}{p}\frac{\bar{A}(f)}{A(f)} . \end{aligned} \tag{6}$$

Similarly, we put

$$\begin{aligned} A(\bar{f}) &= \big\langle \bar{f}\big|H_\mathrm{eff}\big|D^0\big\rangle , \\ \bar{A}(\bar{f}) &= \big\langle \bar{f}\big|H_\mathrm{eff}\big|\overline{D^0}\big\rangle , \\ \bar{\lambda}_{\bar{f}} &= \frac{p}{q}\frac{A(\bar{f})}{\bar{A}(\bar{f})} , \end{aligned} \tag{7}$$

---

[a] Supported in part by NSF of China

[b] e-mail: duds@mail.ihep.ac.cn

where $\bar{f}$ is the CP conjugate state of the final state f, and

$$|\bar{f}\rangle = CP|f\rangle = \eta_{CP}|f\rangle ,$$

where $\eta_{CP} = \pm 1$ is the CP eigenvalue (or CP parity). For the amplitude of $\overline{D^0_p}(t) \to \bar{f}$, we have from (2)

$$\begin{aligned} A\big(\overline{D^0_p}(t) \to \bar{f}\big) &= \langle \bar{f}|H_{\text{eff}}|\overline{D^0_p}(t)\rangle \\ &= \frac{p}{q} g_-(t) A(\bar{f}) + g_+(t)\bar{A}(\bar{f}) \\ &= \bar{A}(\bar{f})\{g_+(t) + \bar{\lambda}_{\bar{f}} g_-(t)\} . \end{aligned} \tag{8}$$

Now it is easy to calculate the time-dependent widths $\Gamma(D^0_p(t) \to f)$ and $\Gamma(\overline{D^0_p}(t) \to \bar{f})$. Using (3), (5) and (8), we have

$$\begin{aligned} \Gamma\big(D^0_p(t) \to f\big) &= \big|A\big(D^0_p(t) \to f\big)\big|^2 \\ &= |A(f)|^2 \{|g_+(t)|^2 + 2\,\mathrm{Re}\left[\lambda_f g^*_+(t) g_-(t)\right] \\ &\quad + |\lambda_f|^2 |g_-(t)|^2\} , \end{aligned} \tag{9}$$

$$\begin{aligned} \Gamma\big(\overline{D^0_p}(t) \to \bar{f}\big) &= |\bar{A}(\bar{f})|^2 \{|g_+(t)|^2 + 2\,\mathrm{Re}\left[\bar{\lambda}_{\bar{f}} g^*_+(t) g_-(t)\right] \\ &\quad + \big|\bar{\lambda}_{\bar{f}}\big|^2 |g_-(t)|^2\} . \end{aligned} \tag{10}$$

In order to compute λ_f and $\bar{\lambda}_{\bar{f}}$, we need first to compute the amplitude ratios $\bar{A}(f)/A(f)$ and $A(\bar{f})/\bar{A}(\bar{f})$. As an example, we consider $D^0, \overline{D^0} \to K^+K^-$. Drawing the decay diagrams (Fig. 1), we see that if we neglect the penguin diagram contribution, the D^0 and $\overline{D^0}$ decay diagrams each involve only one CKM factor, $V_{us}V^*_{cs}$ and $V^*_{us}V_{cs}$, respectively. The only difference of the D^0 and $\overline{D^0}$ decay diagrams is that the initial and final particles change into their CP counterparts. So

$$\begin{aligned} \frac{\bar{A}(f)}{A(f)} &= \frac{\bar{A}(\overline{D^0} \to K^+K^-)}{A(D^0 \to K^+K^-)} \\ &= \eta_{CP}(K^+K^-) \frac{V^*_{us}V_{cs}}{V_{us}V^*_{cs}} = \eta_{CP}(K^+K^-) = +1 . \end{aligned} \tag{11}$$

In (11), V_{cs} and V_{us} are both real in the Wolfenstein parametrization of the CKM matrix, and $\eta_{CP}(f)$ is the CP parity of the final state f. Usually $\eta_{CP} = \pm 1$ for different f. Actually, we can prove that (see the appendix in [8]) if the decays of D^0 and $\overline{D^0}$ only involve one CKM factor respectively, then the ratio $\frac{\bar{A}(f)}{A(f)}$ obeys

$$\frac{\bar{A}(f)}{A(f)} = \eta_{CP}(f) \frac{\mathrm{e}^{-\mathrm{i}\varphi_{\mathrm{wk}}}}{\mathrm{e}^{\mathrm{i}\varphi_{\mathrm{wk}}}} = \eta_{CP}(f) . \tag{12}$$

The last equality holds only for charm decay, because all the CKM matrix elements involved are real if we neglect the penguin contribution.

Define

$$\rho_f = \frac{\bar{A}(f)}{A(f)} , \quad \bar{\rho}_{\bar{f}} = \frac{A(\bar{f})}{\bar{A}(\bar{f})} . \tag{13}$$

From (6) and (7), we have

$$\begin{aligned} \lambda_f &= \frac{q}{p}\rho_f = \eta_{CP}(f)\left|\frac{q}{p}\right| \mathrm{e}^{-\mathrm{i}\varphi} , \\ \bar{\lambda}_{\bar{f}} &= \frac{p}{q}\bar{\rho}_{\bar{f}} = \eta_{CP}(f)\left|\frac{p}{q}\right| \mathrm{e}^{\mathrm{i}\varphi} . \end{aligned} \tag{14}$$

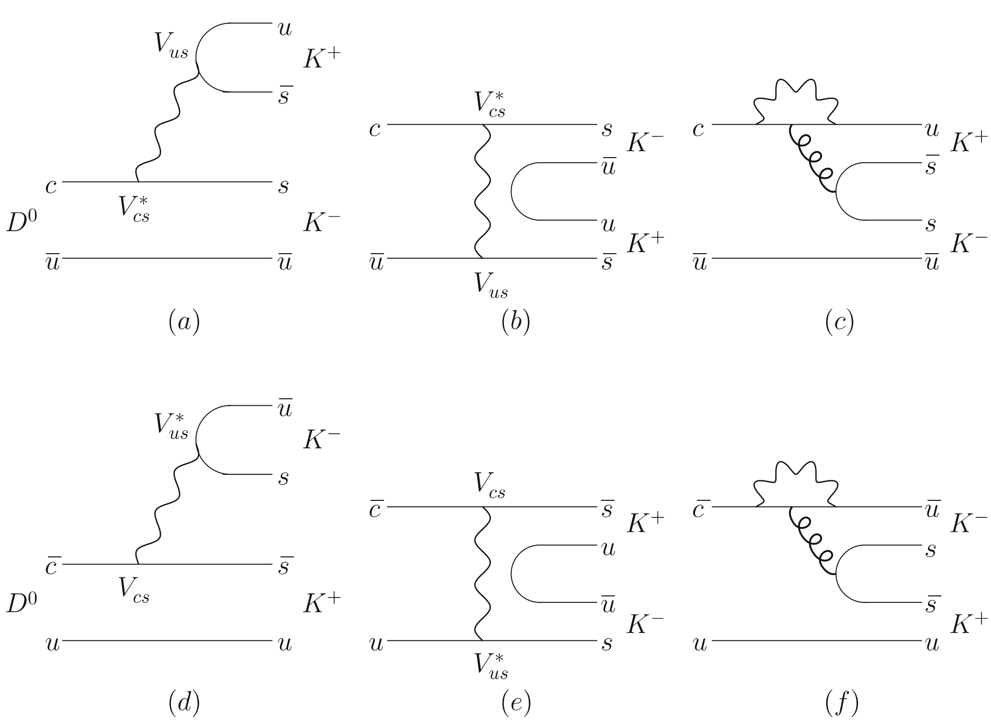

Fig. 1. Decay diagrams for D^0, $\overline{D^0} \to K^+K^-$

After a straightforward calculation we arrive at

$$\begin{aligned}\Gamma\big(D_p^0(t)\to f\big) = {} & \frac{1}{4}\mathrm{e}^{-\gamma t}|A(f)|^2 \\ & \times\Bigg\{\left(1+\left|\frac{q}{p}\right|^2\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right) \\ & +2\left(1-\left|\frac{q}{p}\right|^2\right)\cos\Delta m t \\ & +2\eta_{CP}(f)\left|\frac{q}{p}\right|\Big[\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right)\cos\varphi \\ & +2\sin\varphi\sin\Delta m t\Big]\Bigg\}, \end{aligned} \tag{15}$$

$$\begin{aligned}\Gamma\big(\overline{D_p^0}(t)\to \bar f\big) = {} & \frac{1}{4}\mathrm{e}^{-\gamma t}|\bar A(\bar f)|^2 \\ & \times\Bigg\{\left(1+\left|\frac{p}{q}\right|^2\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right) \\ & +2\left(1-\left|\frac{p}{q}\right|^2\right)\cos\Delta m t \\ & +2\eta_{CP}(f)\left|\frac{p}{q}\right|\Big[\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right)\cos\varphi \\ & -2\sin\varphi\sin\Delta m t\Big]\Bigg\}. \end{aligned} \tag{16}$$

Now we assume

$$|A(f)| = |\bar A(\bar f)|\,. \tag{17}$$

This assumption is guaranteed by our approximation of neglecting the penguin, because in that case only one CKM factor appears [9].

The time-dependent CP asymmetry is

$$C_f(t) = \frac{\Gamma\big(D_p^0(t)\to f\big)-\Gamma\big(\overline{D_p^0}(t)\to \bar f\big)}{\Gamma\big(D_p^0(t)\to f\big)+\Gamma\big(\overline{D_p^0}(t)\to \bar f\big)} \equiv \frac{N_f(t)}{D_f(t)}\,, \tag{18}$$

$$\begin{aligned} N_f(t) = {} & \left(\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right) \\ & -2\left(\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)\cos\Delta m t \\ & +2\eta_{CP}(f)\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}-\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right)\cos\varphi \\ & +4\eta_{CP}(f)\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\sin\varphi\sin\Delta m t\,, \end{aligned} \tag{19}$$

$$\begin{aligned} D_f(t) = {} & \left(2+\left|\frac{q}{p}\right|^2+\left|\frac{p}{q}\right|^2\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right) \\ & +2\left(2-\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)\cos\Delta m t \\ & +2\eta_{CP}(f)\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\left(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}-\mathrm{e}^{\frac{1}{2}\Delta\gamma t}\right)\cos\varphi \\ & +4\eta_{CP}(f)\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\sin\varphi\sin\Delta m t\,. \end{aligned} \tag{20}$$

In (19) and (20), there are several parameters that we need to know: the phase φ, $x=\Delta m/\gamma$, $y=\Delta\gamma/2\gamma$, $|q|/|p|$, etc. But we do know that $|x|\sim|y|\lesssim 10^{-2}$, and that $|q|/|p|$ is very close to unity. Some authors assume [10] that $|q|/|p|-|p|/|q|\lesssim\pm 1\%$. As for the phase φ, we have

$$\frac{q}{p} = \left[\frac{M_{12}^*-\frac{\mathrm{i}}{2}\Gamma_{12}^*}{M_{12}-\frac{\mathrm{i}}{2}\Gamma_{12}}\right]^{1/2} = \left|\frac{q}{p}\right|\mathrm{e}^{-\mathrm{i}\varphi}. \tag{21}$$

In the B^0–$\overline{B^0}$ system, the box diagram dominance leads to

$$\left(\frac{q}{p}\right)_B \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \mathrm{e}^{-2\mathrm{i}\beta}. \tag{22}$$

In the charm case, if we can use the same approximation and assume that the b-quark line plays a dominant role in the corresponding box diagram, then

$$\left(\frac{q}{p}\right)_D \approx \mathrm{e}^{-2\mathrm{i}\gamma}. \tag{23}$$

But this is not the case for charm. Firstly, the box diagram does not dominate. Secondly, even in the box diagram, because $|V_{ub}|$ is very small compared with $|V_{ud}|$ and $|V_{us}|$, the internal b-quark contribution may be unimportant. Furthermore, V_{ud} and V_{us} do not carry the weak phase, so φ gets a contribution from the $\mathrm{i}\Gamma_{12}/2$ term. Anyway, we do not know the value of φ, so we just keep it as a free parameter. For $\mathrm{e}^{\pm\frac{1}{2}\Delta\gamma t}$, using $\Delta\gamma/(2\gamma)=y$, we have $\mathrm{e}^{\pm\frac{1}{2}\Delta\gamma t}=\mathrm{e}^{\pm y\gamma t}=\mathrm{e}^{\pm yt/\tau_{D^0}}$, because $y\lesssim 10^{-2}$, $\mathrm{e}^{\pm yt/\tau_{D^0}}$ is around unity. We have

$$\begin{aligned} N_f(t) \simeq {} & -4\eta_{CP}(f)\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)(y\gamma t)\cos\varphi \\ & +8\eta_{CP}(f)\sin\varphi\sin\Delta m t\,, \end{aligned} \tag{24}$$

$$D_f(t) \simeq 8\,, \tag{25}$$

$$\begin{aligned} C_f(t) & \approx \eta_{CP}(f)\left\{\frac{y\gamma t}{2}\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|\right)\cos\varphi+\sin\varphi\sin\Delta m t\right\} \\ & = \eta_{CP}(f)\left\{\frac{1}{2}y\gamma t\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|\right)\cos\varphi+\sin\varphi\sin(x\gamma t)\right\}. \end{aligned} \tag{26}$$

In [9], the first term in (26) is omitted and the CP parity factor $\eta_{CP}(f)$ is missing.

3 Time-integrated CP asymmetry

In order to have more statistics, we integrate the time-dependent observables over time. We first list some useful quantities:

$$G_+ = \int_0^\infty \mathrm{d}t|g_+(t)|^2 = \frac{2+x^2-y^2}{2\gamma(1+x^2)(1-y^2)} \approx \frac{1}{\gamma}\,, \tag{27}$$

$$G_- = \int_0^\infty \mathrm{d}t|g_-(t)|^2 = \frac{x^2+y^2}{2\gamma(1+x^2)(1-y^2)} \approx \frac{x^2+y^2}{2\gamma}\,, \tag{28}$$

$$\begin{aligned} G_{+-} & = \int_0^\infty \mathrm{d}t g_+^*(t)g_-(t) = \frac{-y(1+x^2)+\mathrm{i}x(1-y^2)}{2\gamma(1+x^2)(1-y^2)} \\ & \approx \frac{-y+\mathrm{i}x}{2\gamma}\,, \end{aligned} \tag{29}$$

for $x^2, y^2 \ll 1$. It is straightforward to get the integrated decay width. From (9) and (10), we have

$$\begin{aligned}\Gamma\left(D_p^0 \to f\right) &= \int_0^\infty \mathrm{d}t \Gamma\left(D_p^0(t) \to f\right) \\ &= |A(f)|^2 \left\{G_+ + 2\,\mathrm{Re}(\lambda_f G_{+-}) + |\lambda_f|^2 G_-\right\},\end{aligned} \tag{30}$$

$$\begin{aligned}\Gamma\left(\overline{D_p^0} \to \bar{f}\right) &= \int_0^\infty \mathrm{d}t \Gamma\left(\overline{D_p^0}(t) \to \bar{f}\right) \\ &= |\bar{A}(\bar{f})|^2 \left\{G_+ + 2\,\mathrm{Re}\left(\bar{\lambda}_{\bar{f}} G_{+-}\right) + \left|\bar{\lambda}_{\bar{f}}\right|^2 G_-\right\}.\end{aligned} \tag{31}$$

Again, if we assume that $|A(f)| = |\bar{A}(\bar{f})|$, then the time-integrated CP asymmetry is

$$C_f = \frac{\Gamma\left(D_p^0 \to f\right) - \Gamma\left(\overline{D_p^0} \to \bar{f}\right)}{\Gamma\left(D_p^0 \to f\right) + \Gamma\left(\overline{D_p^0} \to \bar{f}\right)} \equiv \frac{N_f}{D_f}, \tag{32}$$

$$\begin{aligned}N_f &= -2\eta_{CP}(f)\left[y\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\varphi - x\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\varphi\right] \\ &\quad + (x^2+y^2)\left(\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2\right),\end{aligned} \tag{33}$$

$$\begin{aligned}D_f &= 4 - 2\eta_{CP}(f)\left[y\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\cos\varphi\right. \\ &\quad \left. + x\left(\left|\frac{p}{q}\right| - \left|\frac{q}{p}\right|\right)\sin\varphi\right] + (x^2+y^2)\left(\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2\right) \\ &\approx 4.\end{aligned} \tag{34}$$

Finally, neglecting the (x^2+y^2) term in (33), one may obtain

$$C_f \simeq \eta_{CP}(f)\left\{-\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\varphi + x\sin\varphi\right\}. \tag{35}$$

Up to now, we have only discussed incoherent D^0–$\overline{D^0}$ decays. Sometimes D^0–$\overline{D^0}$ pairs are produced coherently, such as in e^+e^- colliding machines (BES and CLEO-c).

The time-evolved coherent state of a D^0–$\overline{D^0}$ pair can be written as [9]

$$|i\rangle = \left|D^0(k_1,t_1)\overline{D^0}(k_2,t_2)\right\rangle + \eta\left|\overline{D^0}(k_1,t_1)D^0(k_2,t_2)\right\rangle, \tag{36}$$

where η is the charge conjugation parity or the orbital angular momentum parity of the D^0–$\overline{D^0}$ pair.

Because $D^0 \to l^+X$ and $\overline{D^0} \to l^-X$ only, we can use the semileptonic decay to tag one of the two time-evolved states $D_p^0(t)$ and $\overline{D_p^0}(t)$. We define the leptonic-tagging CP asymmetry C_{fl} by

$$C_{fl} = \frac{N(l^-,f) - N(l^+,\bar{f})}{N(l^-,f) + N(l^+,\bar{f})}, \tag{37}$$

where

$$N(l^-,f) = \int_0^\infty \mathrm{d}t_1\,\mathrm{d}t_2 \left|\langle l^-, f|H_{\mathrm{eff}}|\mathrm{i}\rangle\right|^2 \tag{38}$$

is proportional to the number of events in which $D_p^0(k,t) \to l^-X$ is tagging on one side, and the other side is the decay $\overline{D_p^0}(k,t) \to f$ or vice versa. Similarly,

$$N(l^+,\bar{f}) = \int_0^\infty \mathrm{d}t_1\,\mathrm{d}t_2 \left|\langle l^+, \bar{f}|H_{\mathrm{eff}}|\mathrm{i}\rangle\right|^2. \tag{39}$$

Assuming $|A(f)| = |\bar{A}(\bar{f})|$ and $|A(l^+)| = |\bar{A}(l^-)|$, after a tedious calculation, we have

$$\begin{aligned}N(l^-,f) &= |\bar{A}(l^-)A(f)|^2 \\ &\quad \times \left\{G_+^2 + G_-^2 + 2|\lambda_f|^2\left[G_+G_- + \eta|G_{+-}|^2\right]\right. \\ &\quad + 2(1+\eta)G_-\,\mathrm{Re}\left(\lambda_f G_{+-}^*\right) \\ &\quad \left. + 2(1+\eta)G_+\,\mathrm{Re}(\lambda_f G_{+-}) + 2\eta\,\mathrm{Re}\left(G_{+-}^2\right)\right\},\end{aligned} \tag{40}$$

$$\begin{aligned}N(l^+,\bar{f}) &= |A(l^+)\bar{A}(\bar{f})|^2 \\ &\quad \times \left\{G_+^2 + G_-^2 + 2\left|\bar{\lambda}_{\bar{f}}\right|^2\left[G_+G_- + \eta|G_{+-}|^2\right]\right. \\ &\quad + 2(1+\eta)G_-\,\mathrm{Re}\left(\bar{\lambda}_{\bar{f}} G_{+-}^*\right) \\ &\quad \left. + 2(1+\eta)G_+\,\mathrm{Re}\left(\bar{\lambda}_{\bar{f}} G_{+-}\right) + 2\eta\,\mathrm{Re}\left(G_{+-}^2\right)\right\},\end{aligned} \tag{41}$$

$$C_{fl} = \frac{N(l^-,f) - N(l^+,\bar{f})}{N(l^-,f) + N(l^+,\bar{f})} \equiv \frac{N_{fl}}{D_{fl}}, \tag{42}$$

$$\begin{aligned}N_{fl} &= (2+\eta)\frac{x^2+y^2}{2\gamma^2}\left(\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2\right) \\ &\quad + (1+\eta)\eta_{CP}(f)\frac{x^2+y^2}{2\gamma^2} \\ &\quad \times \left[y\left(\left|\frac{p}{q}\right| - \left|\frac{q}{p}\right|\right)\cos\varphi - x\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\varphi\right] \\ &\quad + (1+\eta)\eta_{CP}(f)\frac{1}{\gamma^2} \\ &\quad \times \left[y\left(\left|\frac{p}{q}\right| - \left|\frac{q}{p}\right|\right)\cos\varphi + x\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\varphi\right] \\ &\approx \frac{2(1+\eta)\eta_{CP}(f)}{\gamma^2} \\ &\quad \times \left[-\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\varphi + x\sin\varphi\right],\end{aligned} \tag{43}$$

$$\begin{aligned}D_{fl} &= \frac{2}{\gamma^2} + \frac{(x^2+y^2)^2}{2\gamma^2} \\ &\quad + \frac{1}{2\gamma^2}\left(\left|\frac{p}{q}\right| + \left|\frac{q}{p}\right|\right)(2+\eta)(x^2+y^2) \\ &\quad + (1+\eta)\eta_{CP}(f)\frac{x^2+y^2}{2\gamma^2}\left[-y\left(\left|\frac{p}{q}\right| + \left|\frac{q}{p}\right|\right)\cos\varphi\right. \\ &\quad \left. - x\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\sin\varphi\right] + (1+\eta)\eta_{CP}(f)\frac{1}{\gamma^2} \\ &\quad \times \left[-y\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\cos\varphi - x\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\sin\varphi\right] \\ &\quad + 4\eta\frac{y^2-x^2}{4\gamma^2} \\ &\approx \frac{2}{\gamma^2}.\end{aligned} \tag{44}$$

Table 1. The number of $D\overline{D}$ pairs needed for testing the CP asymmetry

$D^0 \to f$	C_f (theory)	C_f (exp.)	BR	$N_{D\overline{D}}$ (1σ lower bound)
K^+K^-		0.014 ± 0.010	$(3.84 \pm 0.10) \times 10^{-3}$	2.60×10^7
K_sK_s		-0.23 ± 0.19	$(3.7 \pm 0.7) \times 10^{-4}$	2.70×10^8
$K^{*+}K^{*-}$			1.0×10^{-2} (BSW)	1×10^7
$\pi^+\pi^-$	$\lesssim 10^{-3}$	0.013 ± 0.012	$(1.364 \pm 0.032) \times 10^{-3}$	7.4×10^7
$\pi^0\pi^0$		0.00 ± 0.05	$(7.9 \pm 0.8) \times 10^{-4}$	1.26×10^8
$\rho^0\pi^0$			$(3.2 \pm 0.4) \times 10^{-3}$	3.13×10^7
$\rho^+\rho^-$			1.3×10^{-2} (BSW)	7.69×10^6
$\rho^0\rho^0$			1.2×10^{-3} (BSW)	8.33×10^7
$\phi\pi^0$			$(7.4 \pm 0.5) \times 10^{-4}$	1.35×10^8
$\phi\eta$			$(1.4 \pm 0.4) \times 10^{-4}$	7.14×10^8
$K^{*0}K^{*0}$			$(7 \pm 5) \times 10^{-5}$	1.43×10^9

Finally,

$$C_{fl} = \frac{N_{fl}}{D_{fl}} = (1+\eta)\eta_{CP}(f)\left\{ -\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\varphi + x\sin\varphi \right\}. \tag{45}$$

Comparing (45) with (35), we find that C_{fl} is just twice as large as C_f when the charge conjugation parity or the orbital angular momentum l is even. This is surprising. From (35), the order of magnitude of C_f is $\lesssim 10^{-3}$, because $x \sim y \lesssim 10^{-2}$. Now, we present C_f (theory), C_f (exp.), the branching fractions for the $D^0, \overline{D^0}$ decay into CP eigenstates and the number of D–$\overline{D}$ pairs needed for testing the CP asymmetry for a 1σ signal lower bound in Table 1, where we take most of the branching ratios from the 2006 particle data booklet [11]. For the $K^{*+}K^{*-}$, $\rho^+\rho^-$ and $\rho^0\rho^0$ final states, we use the theoretical estimation of Bauer–Stech–Wirbel (BSW) [12]. The measured CP asymmetries listed in Table 1 are also taken from the 2006 particle data group [11]. We use the formula for $N_{D\overline{D}}$:

$$N_{D\overline{D}} = \frac{1}{BC_f^2} \qquad \text{for } 1\sigma \text{ significance};$$

$$N_{D\overline{D}} = \frac{9}{BC_f^2} \qquad \text{for } 3\sigma \text{ significance}.$$

4 Summary and conclusion

We have computed the time-dependent and time-integrated CP asymmetry for the decays of the neutral charmed meson into the CP eigenstates. We present the CP asymmetry not only for incoherent D^0–$\overline{D^0}$, but also for coherent D^0–$\overline{D^0}$ pairs. We find that the time-integrated CP asymmetries are very small (of the order of $\lesssim 10^{-3}$) for testing the CP asymmetries. We also give the lower bound for the number of $D\overline{D}$ pairs needed for testing the CP asymmetries. At present, the integrated luminosities for e^+e^- colliders are

BES II: $27\,\mathrm{pb}^{-1}$,
BES III: $20\,\mathrm{fb}^{-1}$ for four years of data taking,
CLEO-c: $281\,\mathrm{pb}^{-1}$.

The corresponding $D\overline{D}$ pairs are

BES II: 10^5,
BES III: 10^6,
CLEO-c: 10^7.

In Table 1, for the $D \to VV$ decays, only when both vector mesons are longitudinally polarized the VV final states are CP eigenstates [13, 14]. For the corresponding branching ratios in Table 1, we assume that both longitudinally polarized final states dominate. From Table 1 we see that the only hope is to rely on BES III and B-factories. At B-factories, because of the large data sample of charmed mesons, both the time-dependent asymmetry and the time-integrated asymmetry can be measured, while for BES III, only the time-integrated CP asymmetry can be tested. Of course, if there is new physics, some surprise may happen. We can also see from Table 1 that all the measured CP asymmetries presently are consistent with zero.

Acknowledgements. I thank Hai-bo Li, Cai-dian Lü, Mao-zhi Yang and Zhi-zhong Xing for discussions. This work is supported in part by the National Natural Science Foundation of China under grant numbers NSFC 90103011, 10375073 and 90403024.

References

1. A. Datta, D. Kumbhakar, Z. Phys. C **27**, 515 (1985)
2. H. Georgi, Phys. Lett. B **297**, 353 (1992)
3. I.I. Bigi, N.G. Uraltsev, Nucl. Phys. B **592**, 92 (2001)
4. A.F. Falk et al., Phys. Rev. D **65**, 054034 (2002)
5. A.F. Falk et al., Phys. Rev. D **69**, 11402 (2004)

6. I. Shipsey, hep-ex/0607070 (2006)
7. I.I. Bigi, A.I. Sanda, CP Violation (Cambridge Univ. Press, Cambridge, UK, 2001)
8. D.S. Du, I. Dunietz, D.D. Wu, Phys. Rev. D **34**, 3414 (1986)
9. D.S. Du, Phys. Rev. D **34**, 3428 (1986), Equations (11)–(13)
10. Z.Z. Xing, Phys. Rev. D **55**, 196 (1997)
11. http://pdg.lbl.gov, p. 34–44
12. M. Bauer, B. Stech, M. Wirbel, Z. Phys. C **34**, 103 (1989)
13. G. Kramer, W.F. Palmer, Phys. Rev. D **45**, 193 (1992)
14. A.S. Dighe, I. Dunietz, H.J. Lipkin, J.L. Rosner, Phys. Lett. B **369**, 144 (1996)